Revision Algebra 1 & 2

Algebra 1

(a) Formulae
(b) Factorisation
(c) Linear Equations
(d) Simultaneous equations
(e) Quadratic equations
(f) Inequalities

Algebra 2

(a) Algebraic Fractions
(b) Subject of a formula
(c) Direct and Inverse proportion
(d) Indices
(e) Inequalities
(f) Linear Programming
7 Solve the inequality \(3 < 2x - 5 < 7\).

\[
3 + 5 < 2x < 7 + 5 \\
8 < 2x < 12 \\
4 < x < 6
\]

Answer \(4 < x < 6\) \(\ldots\) \(\ldots\) \([2]\)

10 Work out as a single fraction

\[
\frac{2(x+4) - (x-3)}{(x-3)(x+4)} - \frac{1}{x+4}
\]

\[
\frac{2x+8 - x+3}{x^2+x-12}
\]

\[
\frac{x+11}{x^2+x-12}
\]

Answer \(\ldots\) \(\ldots\) \([3]\)
A ferry has a deck area of 3600 m$^2$ for parking cars and trucks. Each car takes up 20 m$^2$ of deck area and each truck takes up 80 m$^2$. On one trip, the ferry carries $x$ cars and $y$ trucks.

(a) Show that this information leads to the inequality $x + 4y \leq 180$.

(b) The charge for the trip is $25 for a car and $50 for a truck. The total amount of money taken is $3000. Write down an equation to represent this information and simplify it.

Answer (b) $x + 2y = 120$  [2]
(c) The line \( x + 4y = 180 \) is drawn on the grid below.

(i) Draw, on the grid, the graph of your equation in part (b).

(ii) Write down a possible number of cars and a possible number of trucks on the trip, which together satisfy both conditions.

50 - 51 cars

29 trucks

Answer (c)(ii) 51 cars, 29 trucks [1]
4. Simplify

\[ \frac{2}{3} p^{12} \cdot \frac{3}{4} p^4 = \frac{1}{2} p^{20} \]

Answer: ................................................................. [2]

5. Solve the equation

\[ \frac{x}{4} - 8 = -2 \]

\[ \frac{x}{4} = 6 \]

\[ x = 24 \quad \text{Answer } x = 24 \] [2]

11. Solve the simultaneous equations

\[ \frac{1}{3} x + y = 5 \]
\[ x - 2y = 6 \]

\[ \begin{align*}
2x + 2y &= 10 \\
x - 2y &= 6 \\
2x &= 16 \\
x &= 8 \\
8 - 2y &= 6 \\
-2y &= 6 - 8 \\
-2y &= -2 \\
y &= 1
\end{align*} \]

Answer \( x = \frac{8}{2} \quad y = 1 \) [3]
13. Make \( d \) the subject of the formula

\[
c = kd^2 + e.
\]

\[
kd^2 = c - e \quad \Rightarrow \quad d^2 = \frac{c - e}{k}
\]

Answer \( d = \sqrt{\frac{c - e}{k}} \) \([3]\)

20. (a) Factorise completely \( 12x^2 - 3y^2 \).

\[
3(4x^2 - y^2) = 3(2x - y)(2x + y)
\]

Answer(a) \( \) \([2]\)

(b) (i) Expand \( (x - 3)^2 \).

\[
(x - 3)^2 = (x - 3)(x - 3) = x^2 - 6x + 9
\]

Answer(b)i) \( x^2 - 6x + 9 \) \([2]\)

(ii) \( x^2 - 6x + 10 \) is to be written in the form \( (x - p)^2 + q \). Find the values of \( p \) and \( q \).

\[
(x - 3)^2 + 1
\]

Answer(b)ii) \( p = \quad q = \) \([2]\)
8 Solve the simultaneous equations

\[ 8 \left( \frac{1}{2}x + 2y = 16 \right) \]
\[ 2 \left( 2x + \frac{1}{2}y = 19 \right) \]

\[ 4x + 4y = 32 \]
\[ 2x + \frac{1}{2}y = 19 \]

\[ 15y = 90 \]
\[ y = 6 \]

Answer \( x = 8 \)

9 The wavelength, \( \lambda \), of a radio signal is inversely proportional to its frequency, \( f \). When \( f = 200 \), \( \lambda = 1500 \).

(a) Find an equation connecting \( f \) and \( \lambda \).

\[ \omega = k \left( \frac{1}{f} \right) \]

\[ 1500 = k \left( \frac{1}{200} \right) \]

\[ k = 300,000 \]

Answer (a) \( \omega = \frac{300,000}{f} \) \[ 2 \]

(b) Find the value of \( f \) when \( \lambda = 600 \).

\[ \lambda = 300,000 \left( \frac{1}{f} \right) \]

\[ f = \frac{300,000}{600} \]

\[ f = 500 \]

Answer (b) \( f = 500 \) \[ 1 \]
16 Simplify \[ \frac{x+2}{x} - \frac{x}{x+2} \].

Write your answer as a fraction in its simplest form.

\[
\frac{(x+2)^2 - x^2}{x(x+2)} = \frac{x^2 + 4x + 4 - x^2}{x(x+2)} = \frac{4x + 4}{x^2 + 2x}.
\]

Answer \[ \frac{4x+4}{x^2 + 2x} \] [3]
19 Solve

(a) \(0.2x + 3.6 = 1.2\).

\[
\begin{align*}
3x + 3.6 &= 12 \\
2x &= 12 - 3.6 \\
2x &= 8.4 \\
x &= -12
\end{align*}
\]

Answer (a) \(x = -12\) \[2\]

(b) \(\frac{2 - 3x}{3} < x + 2\).

\[
\begin{align*}
2 - 3x &< 5x + 10 \\
-8x &< 8 \\
\frac{-8x}{-8} &< \frac{8}{-8} \\
x &> -1
\end{align*}
\]

Answer (b) \[x > -1\] \[3\]

2006

6 Write as a single fraction in its simplest form

\[
\frac{5(x+1) - 4x}{x(x+1)}
\]

\[
\begin{align*}
&= \frac{5x + 5 - 4x}{x(x+1)} \\
&= \frac{x + 5}{x^2 + x}
\end{align*}
\]

Answer \[\frac{x + 5}{x^2 + x}\] \[2\]
12 Solve the simultaneous equations

\[12x + 60y = 360 \]
\[-12x + 200y = 720\]

\[\begin{align*}
-140y &= -420 \\
y &= 3
\end{align*}\]

0.4x + 2y = 10 \quad \rightarrow \quad 4x + 20y = 100 \quad \times 3
0.3x + 5y = 18 \quad \rightarrow \quad 3x + 50y = 180 \quad \times 4

0.4x = 10 \\
x = 25

Answer \ x = \quad \boxed{10}

y = \quad \boxed{3} \quad \text{[3]}

13 Solve the equation

\[\frac{x - 2}{4} = \frac{2x + 5}{3}\]

\[3(x - 2) = 4(2x + 5)\]
\[3x - 6 = 8x + 20\]
\[-5x = 26\]
\[x = -\frac{26}{5}\]

Answer \ x = \quad \boxed{-\frac{26}{5}} \quad \text{[3]}

or \quad -5.2
(a) \(4x^2 - 9\).

\[ \text{Answer(a)} \quad (2x + 3)(2x - 3) \quad [1] \]

(b) \(4x^2 - 9x\).

\[ \text{Answer(b)} \quad \times (4x - 9) \quad [1] \]

(c) \(4x^2 - 9x + 2\).

\[ \text{Answer(c)} \quad (4x - 1)(x - 2) \quad [2] \]
(a) One of the lines in the diagram is labelled $y = mx + c$.
Find the values of $m$ and $c$.

$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{2} = 1$$

Answer: $m = \frac{1}{1}$ \hspace{1cm} $c = \frac{8}{1}$ \hspace{1cm} [1]

(b) Show, by shading all the unwanted regions on the diagram, the region defined by the inequalities

$$x \geq 1, \quad y \leq mx + c, \quad y \geq x + 2 \quad \text{and} \quad y \geq 4.$$

Write the letter $R$ in the region required. \hspace{1cm} [2]
10. Write as a fraction in its simplest form

\[
\frac{(2x-3)^2 + 4^2}{4(x-3)} + \frac{x-3}{4} = \frac{x^2 - 6x + 9 + 16}{4(x-3)} = \frac{x^2 - 6x + 25}{4x - 12}.
\]

12. By shading the unwanted parts of the grid above, find and label the region \( R \) which satisfies the following three inequalities:

\[
y \geq 3, \quad y \geq 5x \quad \text{and} \quad x + y \leq 6.
\]

[3]
13. The quantity $y$ varies as the cube of $(x+2)$.

- When $x = 0$, $y = 32$.
- Find $y$ when $x = 1$.

\[
\begin{align*}
y &= k (x+2)^3 \\
32 &= k (0+2)^3 \\
32 &= k (8) \\
k &= 4
\end{align*}
\]

\[
y = 4 (x+2)^3
\]

\[
y = 4 (1+2)^2 = 4 (3)^2 = 36
\]

\[y = 118\]

*Answer: $y = \ldots$ [3]*

17. (a) $\sqrt{32} = 2^5$. Find the value of $p$.

\[
\begin{align*}
(2^5)^{\frac{1}{2}} &= 2^p \\
2^{\frac{5}{2}} &= 2^p \\
p &= \frac{5}{2}
\end{align*}
\]

*Answer: $p = \ldots$ [2]*

(b) $\sqrt[3]{\frac{1}{8}} = 2^q$. Find the value of $q$.

\[
\begin{align*}
\left(\frac{1}{8}\right)^{\frac{1}{3}} &= 2^q \\
\left(\frac{1}{2^3}\right)^{\frac{1}{3}} &= 2^q \\
\left(2^{-3}\right)^{\frac{1}{3}} &= 2^q
\end{align*}
\]

*Answer: $q = \ldots$ [2]*

2008

2. Simplify

\[
\frac{5}{3} + \frac{5x}{9} - \frac{5x}{18}
\]

**LCM: 18**

\[
\begin{align*}
&= \frac{6x + 10x - 5x}{18} \\
&= \frac{11x}{18}
\end{align*}
\]

*Answer: $\frac{11x}{18}$* [2]
8. Simplify \((27x^3)^{\frac{1}{3}}\).
\[
(3^3 x^3)^{\frac{2}{3}}
\]
\[
(3^2)^{\frac{2}{3}} (x^3)^{\frac{2}{3}}
\]
\[
9x^2
\]

Answer: \(9x^2\)  \[2\]

13. Solve the inequality

\[
\frac{2x-5}{8} > \frac{x+4}{3}
\]
\[
3(2x-5) > (x+4)8
\]
\[
6x-15 > 8x+32
\]
\[
-2x > 47
\]
\[
-2
\]
\[
x < -\frac{47}{2}
\]
\[
x < -23.5
\]

Answer: \(x < -23.5\)  \[3\]
16. Find the co-ordinates of the point of intersection of the straight lines

\[
\begin{align*}
2x + 3y &= 11, \quad \times 3 \\
3x - 5y &= -12, \quad \times 2
\end{align*}
\]

\[
\begin{align*}
6x + 9y &= 33 \\
6x - 10y &= -24
\end{align*}
\]

\[
\begin{align*}
19y &= 57 \\
y &= 3 \\
2x + 3(3) &= 11 \\
6x &= 11 - 9 \\
6x &= 2 \\
x &= \frac{1}{3}
\end{align*}
\]

*Answer* \((\frac{1}{3}, 3)\) [3]

9. Rearrange the formula to make \(y\) the subject.

\[
x + \frac{\sqrt{y}}{9} = 1
\]

\[
\begin{align*}
\frac{\sqrt{y}}{9} &= 1 - x \\
\sqrt{y} &= 9 - 9x \\
y &= (9 - 9x)^2
\end{align*}
\]

*Answer* \(y = \ldots\) [3]

\[
y = 81 - 162x + 81x^2
\]
10 Write \( \frac{1}{c} + \frac{1}{d} - \frac{c-d}{cd} \) as a single fraction in its simplest form.

\[
\frac{d + c - (c-d)}{cd} = \frac{2d}{cd}
\]

Answer \( \frac{2}{c} \) \[3\]

12 Solve the simultaneous equations

\[
\begin{align*}
2y + 3x &= 6, \\
x &= 4y + 16.
\end{align*}
\]

\[
\begin{align*}
2y + 3(4y + 16) &= 6 \quad &x &= 4(-3) + 16 \\
2y + 12y + 48 &= 6 \quad &= -12 + 16 \\
14y &= -42 \quad &= 4 \\
y &= -3
\end{align*}
\]

Answer \( x = \frac{4}{4} \) \[3\]

\( y = \frac{-3}{3} \) \[3\]

13 A spray can is used to paint a wall. The thickness of the paint on the wall is \( t \). The distance of the spray can from the wall is \( d \).

\( t \) is inversely proportional to the square of \( d \).

\[
t = \frac{k \times 1}{d^2}
\]

Find \( t \) when \( d = 8 \).

\[
t = \frac{12.8 \times 1}{8^2} = 0.128
\]

\( k = 12.8 \)

Answer \( t = \frac{1}{d^2} \) \[3\]
(a) Draw the three lines $y = 4$, $2x - y = 4$ and $x + y = 6$ on the grid above.

(b) Write the letter $R$ in the region defined by the three inequalities below.

\[
y \leq 4 \quad 2x - y \geq 4 \quad x + y \geq 6
\]
4. Write as a single fraction \( \frac{\frac{3a}{8} + \frac{4}{5}}{40} \).

\[ \frac{15a + 32}{40} \]

Answer \[ \frac{15a + 32}{40} \] \[ \text{[2]} \]

10. Make \( x \) the subject of the formula.

\[ p = \frac{x + 3}{x} \]

\[ p \cdot x = x + 3 \]

\[ p \cdot x - x = 3 \]

\[ x (p - 1) = 3 \]

\[ x = \frac{3}{p - 1} \]

Answer \( x = \) \[ \text{[4]} \]
By shading the unwanted regions of the grid above, find and label the region \( R \) which satisfies the following four inequalities.

\[
\begin{align*}
y &\geq 2 \\
x + y &
\geq 6 \\
y &\leq x + 4 \\
x + 2y &\leq 18
\end{align*}
\]  

\[ [4] \]

2011

8. \( p \) varies directly as the square root of \( q \).
\( p = 8 \) when \( q = 25 \).

Find \( p \) when \( q = 100 \).

\[
\begin{align*}
p &= k \sqrt{q} \\
p &= k \sqrt{100} \\
p &= 5k \\
\frac{8}{5} &= k
\end{align*}
\]

Answer \( p = \frac{16}{5} \)  

\[ [3] \]
11 Rearrange the formula \( c = \frac{4}{a-b} \) to make \( a \) the subject.

\[
c(a-b) = 4, \quad ac - bc = 4, \quad ac = 4 + bc, \quad a = \frac{4 + bc}{c}
\]

Answer \( a = \) ........................................... [3]

12 Solve the simultaneous equations.

\[
x - 5y = 0 \quad \rightarrow \quad x = 5y
15x + 10y = 17
\]

\[
15(5y) + 10y = 17, \quad x = 5y = 0
7.5y + 10y = 17, \quad x = 5(0.2) = 0
8.5y = 17, \quad x = 1
y = 0.2
\]

Answer \( x = \) ........................................... [3]

\[ y = 0.2 \]
15. Write the following as a single fraction in its simplest form.

\[
\frac{x+1}{x+5} - \frac{x}{x+1} \cdot \frac{(x+1)^3 - x(x+5)}{(x+5)(x+1)} \cdot \frac{x^2 + 2x + 1 - x^2 - 5x}{(x+5)(x+1)} \cdot \frac{-3x + 1}{(x+5)(x+1)}
\]

Answer .................................................. [4]

17. Simplify

(a) \(32x^8 + 8x^2\),

\[
\frac{32x^8}{8x^2} = 4x^{8-2} = 4x^6
\]

Answer(a) ................................................. [2]

(b) \(\left(\frac{x^2}{64}\right)^{\frac{3}{2}}\),

\[
= \frac{x^{3\left(\frac{2}{3}\right)}}{64^{\frac{2}{3}}} = \frac{x^2}{2^4} = \frac{x^2}{16}
\]

Answer(b) ................................................. [2]
**2012**

4

\[ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{p}{12} \]

Work out the value of \( p \).

Show all your working.

\[
12 \left( \frac{3}{5} + \frac{1}{3} + \frac{1}{4} = \frac{p}{12} \right) 12
\]

\[
18 + 4 + 3 = p
\]

\[
p = 25
\]

**Answer \( p = 25 \) \quad [2]**

6

\( x \) is a positive integer and \( 15x - 43 < 5x + 2 \).

Work out the possible values of \( x \).

\[
15x - 43 < 5x + 2
\]

\[
10x < 45
\]

\[
x < 4.5
\]

**Answer \( x \leq 4.5 \) \quad [3]**
11. \( y \) varies directly as the square of \((x - 3)\).
\( y = 16 \) when \( x = 1 \).

Find \( y \) when \( x = 10 \).

\[
y = k (x-3)^2
\]
\[
16 = k (1-3)^2
\]
\[
16 = k (4)
\]
\[
k = 4
\]

\[
y = 4 (x-3)^2
\]
\[
y = 4 (10-3)^2
\]
\[
= 4 (7)^2
\]
\[
= 196
\]

\[\text{Answer } y = 196\] [3]

2013

2. Factorise completely.

\[
kp + 3k + mp + 3m
\]

\[
k(p + 3) + m(p + 3)
\]

\[
(k + m)(p + 3)
\]

\[\text{Answer} \] [2]
12 Solve the equation.

\[5(2y - 17) = 60\]

\[10y - 85 = 60\]

\[10y = 145\]

\[y = 14.5\]

Answer: \(y = 14.5\) [3]

14 \(y\) is inversely proportional to \(x^3\).
\(y = 5\) when \(x = 2\).

Find \(y\) when \(x = 4\).

\[y = \frac{k \times \frac{1}{x^3}}{2} = \frac{k}{8}\]

\[5 = \frac{k}{8}\]

\[k = 40\]

\[y = \frac{40 \times \frac{1}{x^3}}{4^3}\]

\[y = \frac{40}{64}\]

\[y = \frac{5}{8} = 0.625\]

Answer: \(y = 0.625\) [3]
15 Use the quadratic equation formula to solve

\[ 2x^2 + 7x - 3 = 0 \]

Show all your working and give your answers correct to 2 decimal places.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-7 \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}
\]

\[
x = \frac{-7 \pm \sqrt{49 + 24}}{4}
\]

\[
x = \frac{-7 \pm \sqrt{73}}{4}
\]

\[
\left\lfloor \frac{-7 + \sqrt{73}}{4} \right\rfloor \approx 0.39
\]

\[
\frac{-7 - \sqrt{73}}{4} \approx -3.89
\]

Answer \( x = \ldots \) or \( x = \ldots \) [4]
Solve $6x + 3 < x < 3x + 9$ for integer values of $x$.

\[6x + 3 \leq 3x + 9\]

\[2x \leq 6\]

\[x \leq 3\]

\[6x - x \leq 3\]

\[-2x \leq 9\]

\[x \geq -\frac{9}{2}\]

\[5x \leq -3\]

\[x \leq -\frac{3}{5}\]

Answer \[\frac{-9}{2} \leq x \leq -\frac{3}{5}\]

Solution set \[\frac{-9}{2} \leq x \leq -\frac{3}{5}\]