Revision Algebra 1 & 2 – PAPER 4

Algebra 1

(a) Formulae
(b) Factorisation
(c) Linear Equations
(d) Simultaneous equations
(e) Quadratic equations
(f) Inequalities

Algebra 2

(a) Algebraic Fractions
(b) Subject of a formula
(c) Direct and Inverse proportion
(d) Indices
(e) Inequalities
(f) Linear Programming
9 (a) Solve the following equations.

(i) \( \frac{5}{w} = \frac{3}{w+1} \)

Answer(a)(i) \( w = \frac{-2.5}{1} \) [2]

(ii) \( (y + 1)^2 = 4 \)

Answer(a)(ii) \( y = \frac{-3}{1} \) or \( y = \frac{1}{1} \) [2]

(iii) \( \frac{x + 1}{3} - \frac{x - 2}{5} = 2 \)

Answer(a)(iii) \( x = \frac{9, 5}{1} \) [3]

(b) (i) Factorise \( u^2 - 9u - 10 \).

Answer(b)(i) \( (u-10)(u+1) \) [2]

(ii) Solve the equation \( u^2 - 9u - 10 = 0 \).

Answer(b)(ii) \( u = \frac{-1}{1} \) or \( u = \frac{10}{1} \) [1]
The area of the triangle is equal to the area of the square. All lengths are in centimetres.

(i) Show that \( x^2 - 3x - 2 = 0 \).

\[
\frac{(x+1)(x+2)}{2} = x^2
\]

\[
x^2 + 3x + 2 = 2x^2
\]

\[
x^2 - 3x - 2 = 0
\]

(ii) Solve the equation \( x^2 - 3x - 2 = 0 \), giving your answers correct to 2 decimal places. Show all your working.

\[
\text{Answer (c)(ii)} \quad x = -0.56 \quad \text{or} \quad x = 3.56
\]

(iii) Calculate the area of one of the shapes.

\[
12.67 \text{ to } 12.69
\]

\[
\text{Answer (c)(iii)} \quad \ldots \text{ cm}^2
\]
10 A company has a vehicle parking area of 1200 m² with space for x cars and y trucks.

Each car requires 20 m² of space and each truck requires 100 m² of space.

(a) Show that \( x + 5y \leq 60 \).

Answer (a) \[
\frac{30x + 100y}{20} \leq 1200 \\
x + 5y \leq 60
\]

[1]

(b) There must also be space for

(i) at least 40 vehicles,

(ii) at least 2 trucks.

Write down two more inequalities to show this information.

Answer (b)(i) \[ x + y \geq 40 \] [1]

Answer (b)(ii) \[ \frac{x}{5} \geq 2 \] [1]

(c) One line has been drawn for you.

On the grid, show the three inequalities by drawing the other two lines and shading the unwanted regions.
(d) Use your graph to find the largest possible number of trucks.

Answer(d) \[\text{5 trucks} \] [1]

(e) The company charges $5 for parking each car and $10 for parking each truck. Find the number of cars and the number of trucks which give the company the greatest possible income.

Calculate this income.

Answer(e) Number of cars = \[\text{50} \]

Number of trucks = \[\text{2} \]

Greatest possible income = $ \[\text{270} \] [3]
8 (a) $y$ is 5 less than the square of the sum of $p$ and $q$.

Write down a formula for $y$ in terms of $p$ and $q$.

\[ y = \frac{(p + q)^2 - 5}{2} \]

(b) The cost of a magazine is $x$ and the cost of a newspaper is $x(x - 3)$.

The total cost of 6 magazines and 9 newspapers is $51.

Write down and solve an equation in $x$ to find the cost of a magazine.

\[ 6x + 9(x - 3) = 51 \]

\[ 5 \cdot 2 \]

Answer(b) $\$ \dots$ [4]
(c) Bus tickets cost $3 for an adult and $2 for a child.

There are $a$ adults and $c$ children on a bus.

The total number of people on the bus is 52.

The total cost of the 52 tickets is $139.

Find the number of adults and the number of children on the bus.

\[ a + c = 52 \]
\[ 3a + 2c = 139 \]

Answer(s): Number of adults = \[ \frac{35}{5} \]

Number of children = \[ 17 \] [5]
(a) The cost of a bottle of water is $w$.

The cost of a bottle of juice is $j$.

The total cost of 8 bottles of water and 2 bottles of juice is $12.

The total cost of 12 bottles of water and 18 bottles of juice is $45.

Find the cost of a bottle of water and the cost of a bottle of juice.

\[
8w + 2j = 12 \\
12w + 18j = 45
\]

Answer (a) Cost of a bottle of water = $1.05$

Cost of a bottle of juice = $1.2$

(b) Reshni cycles 2 kilometres at $y$ km/h and then runs 4 kilometres at $(y-4)$ km/h. The whole journey takes 40 minutes.

(i) Write an equation in $y$ and show that it simplifies to $y^2 - 13y + 12 = 0$.

Answer (b)(i)

\[
\frac{2}{y} + \frac{4}{y-4} = \frac{40}{60} \quad \text{or better}
\]

\[
y^2 - 13y + 12 = 0
\]
(ii) Factorise \( y^2 - 13y + 12 \).

\[ \text{Answer (b)(ii)} \quad (y-1)(y-12) \quad [2] \]

(iii) Solve the equation \( y^2 - 13y + 12 = 0 \).

\[ \text{Answer (b)(iii)} \quad y = \frac{1}{\text{---}} \quad \text{or} \quad y = \frac{12}{\text{---}} \quad [1] \]

(iv) Work out Roshni’s running speed.

\[ \text{Answer (b)(iv)} \quad \frac{8}{\text{---}} \quad \text{km/h} \quad [1] \]

(c) Solve the equation

\[ u^2 - u - 4 = 0. \]

Show all your working and give your answers correct to 2 decimal places.

\[
-\frac{(-1) \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}
\]

\[ \text{Answer (c)} \quad u = \frac{-1.5\text{c}}{\text{---}} \quad \text{or} \quad u = \frac{2.5\text{c}}{\text{---}} \quad [4] \]
A farmer makes a rectangular enclosure for his animals. He uses a wall for one side and a total of 72 metres of fencing for the other three sides.

The enclosure has width $x$ metres and area $A$ square metres.

(a) Show that $A = 72x - 2x^2$.

\[
72 - 2x \times (72 - 2x) = 72x - 2x^2
\]

\[\text{Answer (a)}\]

(b) Factorise completely $72x - 2x^2$.

\[\text{Answer (b)} \quad 2x (36 - x) \quad \text{or} \quad -2x (x - 36)\]

(c) Complete the table for $A = 72x - 2x^2$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0</td>
<td>310</td>
<td>520</td>
<td>630</td>
<td>640</td>
<td>550</td>
<td>360</td>
<td>70</td>
</tr>
</tbody>
</table>

\[\text{Answer (c)}\]

(d) Draw the graph of $A = 72x - 2x^2$ for $0 \leq x \leq 35$ on the grid opposite.
(e) Use your graph to find

(i) the values of \( x \) when \( A = 450 \),

\[ A = \begin{cases} 7.5 & \text{to} & 8.5 \\ 27.5 & \text{to} & 28.5 \end{cases} \]

\[ x = \quad \text{or} \quad x = \]

(ii) the maximum area of the enclosure.

\[ A = 641 \quad \text{to} \quad 660 \quad \text{m}^2 \]

(f) Each animal must have at least 12 m\(^2\) for grazing.

Calculate the greatest number of animals that the farmer can keep in an enclosure which has an area of 500 m\(^2\).

\[ A = \]

41

\[ \text{Answer} \]

[2]
In the right-angled triangle $ABC$, $AB = x \text{ cm}, BC = (x + 7) \text{ cm}$ and $AC = 17 \text{ cm}$.

(i) Show that $x^2 + 7x - 120 = 0$.

Answer (a)(i)

\[ x^2 + (x + 7)^2 = 17^2 \]

(ii) Factorise $x^2 + 7x - 120$.

Answer (a)(ii) 

\[ (x + 15)(x - 8) \]

(iii) Write down the solutions of $x^2 + 7x - 120 = 0$.

Answer (a)(iii) $x = -15$ or $x = 8$

(iv) Write down the length of $BC$.

Answer (a)(iv) $BC = 15$ cm
(b) 

$$3x \text{ cm} \quad (2x + 3) \text{ cm}$$

$$3x = (2x + 3)^2$$

The rectangle and the square shown in the diagram above have the same area.

(i) Show that $2x^2 - 15x - 9 = 0$.

Answer (b)(i) 

$$3x(2x - 1) = (2x + 3)^2$$

(ii) Solve the equation $2x^2 - 15x - 9 = 0$.

Show all your working and give your answers correct to 2 decimal places.

$$\frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(-9)}}{2(2)}$$

Answer (b)(ii) 

$$x = 8.07 \quad \text{or} \quad x = -0.47$$

(iii) Calculate the perimeter of the square.

Answer (b)(iii) 

$$76.5 \text{ cm}$$
3 (a) Expand the brackets and simplify.

\[ x(x+3)+4x(x-1) \]

Answer (a) \[ 5x^2 + x \text{ or } x(5x-1) \] [2]

(b) Simplify \((3x^3)^2\).

Answer (b) \[ 27x^9 \] [2]

(c) Factorise the following completely.

(i) \[ 7x^2 + 14x^4 \]

Answer (c)(i) \[ 7x^2 \left(1 + 2x^2\right) \] [2]

(ii) \[ xy + xw + 2ay + 2aw \]

Answer (c)(ii) \[ (y+w)(x+2a) \] [2]

(iii) \[ 4x^2 - 49 \]

Answer (c)(iii) \[ (2x+7)(2x-7) \] [1]
(d) Solve the equation.

\[ 2x^2 + 5x + 1 = 0 \]

Show all your working and give your answers correct to 2 decimal places.

\[
\frac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{2(2)}
\]

Answer (d) \( x = -2.2 \) or \( x = -0.22 \) [4]
3 (a) $p$ varies inversely as $(m + 1)$.  
When $p = 4$, $m = 8$.  
Find the value of $p$ when $m = 11$.  
\[ p = \frac{k}{m + 1} \]  
\[ k = 3 \times 4 \]  
\[ \text{Answer(a)} \quad p = \frac{3 \times 4}{11 + 1} = \frac{12}{12} = 1 \]  

(b) (i) Factorise $x^2 - 25$.  
\[ \text{Answer(b)(i)} \quad (x + 5)(x - 5) \]  

(ii) Simplify $\frac{2x^2 + 11x + 5}{x^2 - 25}$.  
\[ \text{Answer(b)(ii)} \quad \frac{2x + 1}{(x + 5)(x - 5)} \]  

(c) Solve the inequality $5(x - 4) < 3(12 - x)$.  
\[ \text{Answer(c)} \quad x < 7 \]
9 Peter wants to plant $x$ plum trees and $y$ apple trees.

He wants at least 3 plum trees and at least 2 apple trees.

(a) Write down one inequality in $x$ and one inequality in $y$ to represent these conditions.

\[ x \geq 3, \quad y \geq 2 \] \hspace{1cm} [2]

(b) There is space on his land for no more than 9 trees.

Write down an inequality in $x$ and $y$ to represent this condition.

\[ x + y \leq 9 \] \hspace{1cm} [1]

(c) Plum trees cost $6 and apple trees cost $14.

Peter wants to spend no more than $84.

Write down an inequality in $x$ and $y$, and show that it simplifies to $3x + 7y \leq 42$.

\[ 6x + 14y \leq 84 \] \hspace{1cm} [1]
(d) On the grid, draw four lines to show the four inequalities and shade the unwanted regions.

(e) Calculate the smallest cost when Peter buys a total of 9 trees.

Answer(e) $70$
2 (a) Find the integer values for \( x \) which satisfy the inequality 

\[-3 < 2x - 1 < 6.\]

\[\text{Answer(a)} \quad 0, 1, 2, 3 \quad [3]\]

(b) Simplify \( \frac{x^2 + 3x - 10}{x^2 - 25} \).

\[\text{Answer(b)} \quad \frac{x - 2}{x - 5} \quad [4]\]

(c) (i) Show that \( \frac{5}{x - 3} + \frac{2}{x + 1} = 3 \) can be simplified to \( 3x^2 - 13x - 8 = 0 \).

\[\text{Answer(c)(i)} \quad 5(x + 1) + 2(x - 3) = 3(x + 1)(x - 3) \quad [3]\]

(ii) Solve the equation \( 3x^2 - 13x - 8 = 0 \).

Show all your working and give your answers correct to two decimal places.

\[-\frac{(-13) \pm \sqrt{(-13)^2 - 4(3)(-8)}}{2(3)}\]

\[\text{Answer(c)(ii)} x = 4.88 \quad \text{or} \quad x = -0.55 \quad [4]\]
8 Mr Chang hires \( x \) large coaches and \( y \) small coaches to take 300 students on a school trip. Large coaches can carry 50 students and small coaches 30 students. There is a maximum of 5 large coaches.

(a) Explain clearly how the following two inequalities satisfy these conditions.

(i) \( x \leq 5 \)

Answer (a)(i) \[ \text{There are up to 5 large coaches} \] [1]

(ii) \( 5x + 3y \geq 30 \)

Answer (a)(ii) \[ 50x + 30y \geq 300 \] [2]

Mr Chang also knows that \( x + y \leq 10 \).

(b) On the grid, show the information above by drawing three straight lines and shading the unwanted regions.

[Diagram of a grid with lines and shading] [5]
(c) A large coach costs $450 to hire and a small coach costs $350.

(i) Find the number of large coaches and the number of small coaches that would give the minimum hire cost for this school trip.

Answer (c)(i) Large coaches .......................... 5
Small coaches .......................... 2 .......................... [2]

(ii) Calculate this minimum cost.

Answer (c)(ii) $ .......................... 2950 .......................... [1]
6 (a) A parallelogram has base \((2x - 1)\) metres and height \((4x - 7)\) metres. The area of the parallelogram is 1 m\(^2\).

(i) Show that \(4x^2 - 9x + 3 = 0\).

*Answer (a)(i)* \((4x - 7)(2x - 1) = 1\)

(ii) Solve the equation \(4x^2 - 9x + 3 = 0\).

Show all your working and give your answers correct to 2 decimal places.

\[
-x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(4)(3)}}{2(4)}
\]

*Answer (a)(ii)* \(x = \frac{0.41}{1.84} \) or \(x = \frac{1.84}{0.41} \)

(iii) Calculate the height of the parallelogram.

\(0.36\) or \(0.3720\) to \(0.3724\)

*Answer (a)(iii)* \(0.37\) m
(b) (i) Factorise \( x^2 - 16. \)

\[ \text{Answer (b)(i)} \quad (x - 4)(x + 4) \quad [1] \]

(ii) Solve the equation \( \frac{2x + 3}{x - 4} + \frac{x + 40}{x^2 - 16} = 2. \)

\[ \text{Answer (b)(ii)} \quad x = 7 \quad [4] \]
(a) Solve the equations.

(i) \(4x - 7 = 8 - 2x\)

\[ Answer(a)(i) x = \frac{2.5 \text{ or } \frac{5}{2}}{2} \]

(ii) \(\frac{x - 7}{3} = 2\)

\[ Answer(a)(ii) x = \frac{13}{2} \]

(b) Simplify the expressions.

(i) \((3xy^4)^3\)

\[ Answer(b)(i) 27x^3y^{12} \]

(ii) \((16a^3b^2)^{\frac{1}{2}}\)

\[ Answer(b)(ii) 4a^3b \]

(iii) \(\frac{x^2 - 7x - 8}{x^2 - 64}\)

\[ Answer(b)(iii) \frac{x + 1}{x + 8} \]

[4]
5. Paul buys a number of large sacks of fertiliser costing $x$ each.

He spends $27.

(a) Write down, in terms of $x$, an expression for the number of large sacks which Paul buys.

\[
\frac{\frac{27}{x}}{} \quad \text{Answer(a)} \quad \text{[1]}
\]

(b) Rula buys a number of small sacks of fertiliser.

Each small sack costs $2 less than a large sack.

Rula spends $25.

Write down, in terms of $x$, an expression for the number of small sacks which Rula buys.

\[
\frac{\frac{25}{x-2}}{} \quad \text{Answer(b)} \quad \text{[1]}
\]

(c) Rula buys 4 more sacks than Paul.

Write down an equation in $x$ and show that it simplifies to $2x^2 - 3x - 27 = 0$.

\[
\frac{\frac{25}{x-2}}{} - 4 = \frac{\frac{27}{x}}{} \quad \text{Answer(c)} \quad \text{[4]}
\]

(d) Solve $2x^2 - 3x - 27 = 0$.

\[
\text{Answer(d) } x = \frac{-3}{2} \quad \text{or} \quad x = \frac{4.5}{2} \quad \text{[3]}
\]

(e) Calculate the number of sacks which Paul buys.

\[
\text{Answer(e)} \quad \text{[1]}
\]
10  (a) Write as a single fraction

(i) \( \frac{5}{4} - \frac{2x}{3} \),

\[ \frac{35 - 8x}{20} \]

Answer (a)(i) ................................................. [2]

(ii) \( \frac{4}{x + 3} + \frac{2x - 1}{3} \).

Answer (a)(ii) .................................................. [3]

(b) Solve the simultaneous equations.

\[ 9x - 2y = 12 \]
\[ 3x + 4y = -10 \]

Answer (b) \( \frac{2}{3} \) or 0.667

\[ x = \frac{2}{3} \]

\[ y = -3 \] [3]
(c) Simplify \( \frac{7x + 21}{2x^2 + 9x + 9} \)

Answer (c) \( \frac{7}{2x+3} \) [4]
(a) Solve the equation \( 8x^2 - 11x - 11 = 0 \).
Show all your working and give your answers correct to 2 decimal places.

\[ \begin{align*}
\text{Answer (a)} & \quad x = -0.67 \quad \text{or} \quad x = 2.05 \quad [4] \\
\end{align*} \]

(b) \( y \) varies directly as the square root of \( x \).
\( y = 18 \) when \( x = 9 \).

Find \( y \) when \( x = 484 \).

\[ \begin{align*}
\text{Answer (b)} & \quad y = \frac{132}{x} \quad [3] \\
\end{align*} \]
(e) Sura spends $x on pens which cost $2.50 each. She also spends $x - 14.50 on pencils which cost $0.50 each. The total of the number of pens and the number of pencils is 19.

Write down and solve an equation in $x$.

Answer(s) $x = \frac{20}{6}$
(a) Find the equations of the lines $L_1$, $L_2$, and $L_3$.

Answer (a)

$L_1$ $y = 2$

$L_2$ $y = \frac{2}{3}x$  

$L_3$ $y = -\frac{1}{2}x + 5$  [5]

(b) Write down the three inequalities that define the shaded region, $R$.

Answer (b)

$y > 2$

$y \leq \frac{2}{3}x$

$y \leq -\frac{1}{2}x + 5$  [5]
(e) A gardener buys \( x \) bushes and \( y \) trees.

The cost of a bush is $30 and the cost of a tree is $200.

The shaded region \( R \) shows the only possible numbers of bushes and trees the gardener can buy.

(i) Find the number of bushes and the number of trees when the total cost is $720.

\[
\begin{align*}
\text{Answer (c)(i)} & : 4 \quad \text{bushes} \\
& : 3 \quad \text{trees} \quad [2]
\end{align*}
\]

(ii) Find the number of bushes and the number of trees which give the greatest possible total cost.

Write down this greatest possible total cost.

\[
\begin{align*}
\text{Answer (c)(ii)} & : 2 \quad \text{bushes} \\
& : 4 \quad \text{trees} \\
\text{Greatest possible total cost} & = \$ \quad 860 \quad [3]
\end{align*}
\]
2 (a) Rearrange the formula \( v^2 = u^2 - 2as \) to make \( u \) the subject.

\[
\begin{align*}
\text{Answer (a) } u &= \frac{\pm\sqrt{v^2 + 2as}}{2} \quad [2]
\end{align*}
\]

(b) Chuck cycles along Skyline Drive.
He cycles 60 km at an average speed of \( x \) km/h.
He then cycles a further 45 km at an average speed of \( (x + 4) \) km/h.
His total journey time is 6 hours.

(i) Write down an equation in \( x \) and show that it simplifies to \( 2x^2 - 27x - 80 = 0 \).

\[
\frac{60}{x} + \frac{45}{x + 4} = 6 \quad [4]
\]

(ii) Solve \( 2x^2 - 27x - 80 = 0 \) to find the value of \( x \).

\[
\text{Answer (b)(ii) } x = 16 \quad [3]
\]
4. (a) Expand and simplify.

(i) \(4(2x - 1) - 3(3x - 5)\)

Answer: \(11 - x\) [2]

(ii) \((2x - 3y)(3x + 4y)\)

Answer: \(6x^2 - xy - 12y^2\) [3]

(b) Factorise.

\(x^2 - 5x\)

Answer: \(x(x - 5)\) [1]

(c) Solve the inequality.

\[\frac{2x + 1}{3} \leq \frac{5x - 8}{4}\]

\(x \geq 4\) or \(4 \leq x\)

Answer: \(x \geq 4\) [3]
(d) (i) \[x^2 - 9x + 12 = (x-p)^2 - q\]

Find the value of \(p\) and the value of \(q\).

\[
\begin{align*}
\text{Answer (d)(i)} 
\quad p &= 8.5 \\
\quad q &= 2.5 \\
\end{align*}
\]

[3]

(ii) Write down the minimum value of \(x^2 - 9x + 12\).

\[
\begin{align*}
\text{Answer (d)(ii)} 
\quad \text{minimum value} &= -8.25 \\
\end{align*}
\]

[1]

(iii) Write down the equation of the line of symmetry of the graph of \(y = x^2 - 9x + 12\).

\[
\begin{align*}
\text{Answer (d)(iii)} 
\quad \text{equation} &= 4.5 \\
\end{align*}
\]

[1]