MATRICES and TRANSFORMATION

Paper 2
November 2014

14

\[ A = \begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix} \]

Work out \( A^2 - 4A \).

\[
\begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix} - 4 \begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 16 \\ 4 & 16 \end{pmatrix} - \begin{pmatrix} 8 & 32 \\ 4 & 16 \end{pmatrix} = \begin{pmatrix} 4 & 16 \\ 2 & 8 \end{pmatrix}
\]

Answer \[ \begin{pmatrix} 4 & 16 \\ 2 & 8 \end{pmatrix} \] [3]

3

Write down the order of rotational symmetry of this shape.

Answer \[ 8 \] [1]

11

\[ A = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 \\ -5 & 7 \end{pmatrix} \]

(a) Calculate \( BA \).

\[
\begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -5 & 7 \end{pmatrix} = \begin{pmatrix} 6 & -4 \\ -8 & 38 \end{pmatrix}
\]

Answer(a) \( BA = \begin{pmatrix} 6 & -4 \\ -8 & 38 \end{pmatrix} \) [2]

(b) Find the determinant of \( A \).

\[
\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc
\]

Answer(b) \[ 14 \] [1]

\[
\det(A) = 10 - 1(-2) = 12 + 2 = 14
\]
19 (a) \[ N = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \]

Describe fully the single transformation represented by \( N \).

**Answer (a)** rotation 90° clockwise about origin

---

(b) Find the matrix which represents the single transformation that maps triangle \( A \) onto triangle \( B \).

**Answer (b)** \[ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]

---

(c) On the grid, draw the image of triangle \( A \) under a stretch, factor 3, with the y-axis invariant.

**Answer (c)** [Diagram]
June 2014

15. \[ M = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix} \]

Find

(a) \[ M^2 = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 22 & 18 \\ 27 & 31 \end{pmatrix} \]

(b) the determinant of \( M \).

\[ |M| = ac - bd \]
\[ = 30 + 6 \]
\[ = 36 \]

18. \( A = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix} \)

(a) Calculate \( A^2 \).

\[ A^2 = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 33 & 14 \\ 32 & 17 \end{pmatrix} \]

(b) Calculate \( A^{-1} \), the inverse of \( A \).

\[ A^{-1} = \frac{1}{ac - bd} \begin{pmatrix} c & -b \\ -c & a \end{pmatrix} \]

For \( A = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix} \)

\[ A^{-1} = \frac{1}{15 - 8} \begin{pmatrix} 3 & -2 \\ -4 & 5 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 3 & -2 \\ -4 & 5 \end{pmatrix} \]
(a) Draw the image of triangle A after a translation by the vector \( \begin{pmatrix} 3 \\ -4 \end{pmatrix} \). [2]

(b) Describe fully the single transformation which maps triangle A onto triangle B.

Answer(b) \[ \text{Rotation centered at } (1,0), \ 180^\circ \text{ or half turn} \]

Other answer \[ \text{enlargement centered at } (0,0), \ \text{scale factor of } -1 \] [3]

(c) Draw the image of triangle A after the transformation represented by the matrix \( \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix} \). [3]
November 2013

5  (a) Add one line to the diagram so that it has two lines of symmetry.

(b) Add two lines to the diagram so that it has rotational symmetry of order 2.

Possible Answers:

[Diagrams of possible answers are shown here]
\[ M = \begin{pmatrix} 2 & 1 \\ 4 & 6 \end{pmatrix}, \quad N = \begin{pmatrix} 5 & 0 \\ 1 & 5 \end{pmatrix} \]

(a) Work out \( MN \).

Answer (a) \( MN = \begin{pmatrix} 11 & 5 \\ 26 & 30 \end{pmatrix} \) \[ \square \]

(b) Find \( M^{-1} \).

\[ M^{-1} = \frac{1}{16 - 4} \begin{pmatrix} 6 & -4 \\ -1 & 2 \end{pmatrix} \]

\[ = \frac{1}{8} \begin{pmatrix} 6 & -4 \\ -1 & 2 \end{pmatrix} \]

Answer (b) \( M^{-1} = \) \[ \square \]
\[ A = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]

Work out the following.

(a) \[ AI = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} \]

Answer (a) \( AI = \) \[
\]

(b) \[ A^{-1} \]

\[ A^{-1} = \frac{1}{c - (-4)} \begin{pmatrix} 2 & 1 \\ -4 & 3 \end{pmatrix} = \frac{1}{10} \begin{pmatrix} 2 & 1 \\ -4 & 3 \end{pmatrix} \]

Answer (b) \( A^{-1} = \) \[
\]
17 \((p, q)\) is the image of the point \((x, y)\) under this combined transformation.

\[
\begin{pmatrix}
p \\ q
\end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}
\]

(a) Draw the image of the triangle under the combined transformation.

(b) Describe fully the single transformation represented by \(\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\).

Answer (b) \hspace{1cm} \text{reflection in } y\text{axis} \hspace{1cm} [2]
June 2013

24. \( \begin{pmatrix} \frac{1}{3} & 2 \\ 4 & \frac{1}{2} \end{pmatrix} \) \text{ B } = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}

Find

(a) \( AB \).

\[ \text{Answer(a) } AB = \begin{pmatrix} 6 & 7 \\ 16 & 17 \end{pmatrix} \] [2]

(b) \( B^{-1} \), the inverse of \( B \).

\[ \text{Answer(b) } B^{-1} = \frac{1}{5} \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix} \] [2]

The shaded shape has rotational symmetry of order 2.

Work out the shaded area.

\[ 2 \left[ 4 \times 10 + (18 \times 5) \right] \]

\[ = 5 \times 130 \]

\[ = 650 \]

\[ \text{Answer } \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ddots \text{ cm}^2 \] [3]
17 \[ \mathbf{M} = \begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix}, \quad \mathbf{N} = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 7 & 2 \end{pmatrix} \]

(a) Work out \( \mathbf{MN} \).

\[ \text{Answer (a)} \quad \begin{pmatrix} 7 & 23 & 14 \\ 12 & 41 & 27 \end{pmatrix} \quad [2] \]

(b) Find \( \mathbf{M}^{-1} \), the inverse of \( \mathbf{M} \).

\[ \text{Answer (b)} \quad \frac{1}{3} \begin{pmatrix} 6 & -3 \\ -3 & 2 \end{pmatrix} \quad [2] \]

November 2012

13 Find the matrix which represents the combined transformation of a reflection in the \( x \) axis followed by a reflection in the line \( y = x \).

\[ \text{Answer} \quad \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad [3] \]
\[ M = \begin{pmatrix} 5 & -4 \\ 2 & 3 \end{pmatrix} \]

Find

(a) \( M^T \).

\[ \text{Answer(a)} \quad \begin{pmatrix} 17 & -32 \\ 14 & 1 \end{pmatrix} \quad [2] \]

(b) \( 2M \).

\[ \text{Answer(b)} \quad \begin{pmatrix} 10 & -8 \\ 4 & 6 \end{pmatrix} \quad [1] \]

(c) \( |M| \), the determinant of \( M \).

\[ \text{Answer(c)} \quad \frac{23}{23} \quad [1] \]

(d) \( M^{-1} \).

\[ \text{Answer(d)} \quad \frac{1}{23} \begin{pmatrix} 3 & 4 \\ -2 & 5 \end{pmatrix} \quad [2] \]
The triangle $PQR$ has co-ordinates $P(-1, 1), Q(1, 1)$ and $R(1, 2)$.

(a) Rotate triangle $PQR$ by $90^\circ$ clockwise about $(0, 0)$. Label your image $P'Q'R'$.

(b) Reflect your triangle $P'Q'R'$ in the line $y = -x$. Label your image $P''Q''R''$.

(c) Describe fully the single transformation which maps triangle $PQR$ onto triangle $P''Q''R''$.

Answer(s) reflection in the $x$-axis
(a) Describe the single transformation which maps $ABCD$ onto $A'B'C'D'$.

\textbf{Answer (a)} \hspace{1cm} \text{shear at x-axis: invariant sf 3} \hspace{1cm} [3]

(b) A single transformation maps $A'B'C'D'$ onto $A''B''C''D''$. Find the matrix which represents this transformation.

\textbf{Answer (b)} \hspace{1cm} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \hspace{1cm} [2]

$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ \hspace{1cm} $B = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

On the grid on the next page, draw the image of $PQRS$ after the transformation represented by $BA$. 

\[\text{[5]}\]
12 (a) \[ \mathbf{M} = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix} \]

Find \( \mathbf{M}^{-1} \), the inverse of \( \mathbf{M} \).

\[ \frac{1}{5} \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} \]  [2]

(b) \( \mathbf{D}, \mathbf{E} \) and \( \mathbf{X} \) are \( 2 \times 2 \) matrices.
\( \mathbf{I} \) is the identity \( 2 \times 2 \) matrix.

(i) Simplify \( \mathbf{D} \mathbf{I} \).
\[ \mathbf{D} \mathbf{I} = \mathbf{D} \]

Answer (b)(i) \[ \mathbf{D} \]  [1]

(ii) \( \mathbf{D} \mathbf{X} = \mathbf{E} \)
Write \( \mathbf{X} \) in terms of \( \mathbf{D} \) and \( \mathbf{E} \).
\[ \mathbf{X} = \mathbf{D}^{-1} \mathbf{E} \]

Answer (b)(ii) \( \mathbf{X} = \mathbf{D}^{-1} \mathbf{E} \)  [1]
(a) Draw the image when triangle $A$ is reflected in the line $x = 0$.  

(b) Draw the image when triangle $A$ is rotated through $90^\circ$ anticlockwise about $(-4, 0)$.  

(c) (i) Describe fully the single transformation that maps triangle $A$ onto triangle $B$.  

Answer(c)(i) \hspace{1cm} \text{enlargement scale factor of } 2 \hspace{1cm} \text{centre } (-4, 7) \hspace{1cm} [3]  

(ii) Complete the following statement.  

Area of triangle $A$ : Area of triangle $B$ = \[
\frac{1}{3} : \frac{4}{12} \quad \text{or} \quad \frac{1}{4} : 1
\]  

[2]
(d) Write down the matrix that represents a stretch, factor 4 with the y-axis invariant.

\[
\text{Answer (d)} \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \quad [2]
\]

(e) (i) On the grid, draw the image of triangle \( \Delta \) after the transformation represented by the matrix \( \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \).

(ii) Describe fully this single transformation.

\[
\text{Answer (e)(ii) shear y-axis or x=0 invariant} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} [2]
\]

(iii) Find the inverse of the matrix \( \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \).

\[
\text{Answer (e)(iii)} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \quad [2]
\]
(a) Describe fully the single transformation that maps triangle $A$ onto triangle $B$.

Answer:  
Enlargement $\sqrt{SF} = 1 \sqrt{2}$  
Centre: $(2, 1)$  

[3]
(b) On the grid, draw the image of

(i) triangle \( A \) after a reflection in the line \( x = -3 \),

(ii) triangle \( A \) after a rotation about the origin through \( 270^\circ \) anticlockwise,

(iii) triangle \( A \) after a translation by the vector \( \begin{pmatrix} -1 \\ -2 \end{pmatrix} \)

(c) \( M \) is the matrix that represents the transformation in part (b)(ii).

(i) Find \( M \).

\[
M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
\]

(ii) Describe fully the single transformation represented by \( M^{-1} \), the inverse of \( M \).

Answer(c)(ii) \( \text{rotation } 90^\circ \text{ (anticlockwise)} \) \( \text{from origin} \)
\[ P = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} -3 \\ 5 \end{pmatrix} \]

(a) Work out

(i) \(4P\),

Answer (a)(i) \( \begin{pmatrix} 0 & -4 \\ 4 & 0 \end{pmatrix} \) [1]

(ii) \(P - Q\),

Answer (a)(ii) \( \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \) [1]

(iii) \(P^2\),

Answer (a)(iii) \( \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \) [2]

(iv) \(QR\),

Answer (a)(iv) \( \begin{pmatrix} -13 \\ 5 \end{pmatrix} \) [2]

(b) Find the matrix \(S\), so that \(QS = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\).

Answer (b) \( \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \) [3]
June 2014

1. \( A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix} \quad B = (-2 \ 5) \quad C = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \)

(a) Work out, when possible, each of the following.
If it is not possible, write ‘not possible’ in the answer space.

(i) 2A

\[ \text{Answer(a)(i)} \begin{pmatrix} 6 & 8 \\ -2 & 2 \end{pmatrix} \]  [1]

(ii) \( B + C \)

\[ \text{Answer(a)(ii)} \text{ not possible} \]  [1]

(iii) \( AD \)

\[ \text{Answer(a)(iii)} \begin{pmatrix} 6 & 4 \\ -2 & 2 \end{pmatrix} \]  [2]

(iv) \( A^{-1} \), the inverse of A.

\[ \text{Answer(a)(iv)} \frac{1}{5} \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix} \]  [2]

(b) Explain why it is not possible to work out CD.

\[ \text{Answer(b)} \text{ column in C and rows in D} \]  [1]

(c) Describe fully the single transformation represented by the matrix \( D \).

\[ \text{Answer(c)} \text{ enlargement \ [factor\ 2 \ [centre\] \( (0,0) \)}} \]  [3]
(a) On the grid.

(i) draw the image of shape \( A \) after a translation by the vector \( \begin{pmatrix} -5 \\ -4 \end{pmatrix} \). [2]

(ii) draw the image of shape \( A \) after a rotation through 90° clockwise about the origin. [2]

(b) (i) On the grid, draw the image of shape \( A \) after the transformation represented by the matrix \( \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \).

(ii) Describe fully the single transformation represented by the matrix \( \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \).

\[ \text{Answer: (ii)} \]

\[ \begin{array}{c}
\text{stretch } [\text{factor } 2] \\
\text{invariant line } y \text{-axis } \end{array} \] [3]
(a) Draw the reflection of shape $Q$ in the line $x = -1$.

(b) (i) Draw the enlargement of shape $Q$, centre $(0, 0)$, scale factor $-2$.

(ii) Find the $2 \times 2$ matrix that represents an enlargement, centre $(0, 0)$, scale factor $-2$.

Answer (ii): \[
\begin{pmatrix}
-2 & 0 \\
0 & -2
\end{pmatrix}
\]
(c) (i) Draw the stretch of shape $Q$, factor 2, $x$-axis invariant.

(ii) Find the $2 \times 2$ matrix that represents a stretch, factor 2, $x$-axis invariant.

\[
\text{Answer (c)(ii) } \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}
\]

(iii) Find the inverse of the matrix in part (c)(ii).

\[
\text{Answer (c)(iii) } \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}
\]

(iv) Describe fully the single transformation represented by the matrix in part (c)(iii).

\[
\text{Answer (c)(iv) Stretch factor } \left( \frac{1}{2} \right) \text{ invariant line } x \text{-axis}
\]

\[
\text{Answer (c)(iv) Stretch factor } \left( \frac{1}{2} \right) \text{ invariant line } x \text{-axis}
\]
(i) Draw the reflection of triangle $T$ in the line $y = 5$. [2]

(ii) Draw the rotation of triangle $T$ about the point $(4, 2)$ through $180^\circ$. [2]

(iii) Describe fully the single transformation that maps triangle $T$ onto triangle $U$.

$Answer (a)(iii) \text{ shear } x\text{-axis is invariant, } \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \text{ } [3]$

(iv) Find the $2 \times 2$ matrix which represents the transformation in part (a)(iii).

$Answer (a)(iv) \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \text{ } [2]$
(a) Describe fully the single transformation that maps triangle $A$ onto

(i) triangle $B$,

Answer(a)(i) reflection $x = -2$ oe

(ii) triangle $C$,

Answer(a)(ii) translation $(\frac{-7}{2})$ oe

(iii) triangle $D$.

Answer(a)(iii) stretch $x$-axis oe invariant [factor] $3$
(b) On the grid, draw

(i) the rotation of triangle $A$ about $(6, 0)$ through $90^\circ$ clockwise,  
(ii) the enlargement of triangle $A$ by scale factor $-2$ with centre $(0, -1)$, 
(iii) the shear of triangle $A$ by shear factor $-2$ with the $y$-axis invariant.

(c) Find the matrix that represents the transformation in part (b)(iii).

\[ \text{Answer(c)} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \] [2]